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Erratum

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Volume **11**, Number 2, June (1974), in the article entitled "A Note on Rational Chebyshev Approximation on the Positive Real Axis," by A. R. Reddy, pp. 201–202.

The proof given to the following theorem is valid only in the case $\tau = \omega$.

THEOREM. *Let* $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k \geq 0$ $(k \geq 0)$, *be any entire function of order* ρ $(0 < \rho < \infty)$, *type* τ *and lower type* ω $(0 < \omega \leq \tau < \infty)$. *Then one cannot find algebraic polynomials* $P_n(x)$ *and* $Q_n(x)$ *with nonnegative real coefficients and of degree at most n for which*

$$\lim_{n\to\infty} \left\| \frac{1}{f(x)} - \frac{P_n(x)}{Q_n(x)} \right\|_{L_\infty[0,\infty)}^{1/n} < (2\sqrt{2})^{-\tau/(\rho\omega)}. \tag{1}$$

Even though the approach is fundamentally correct, calculations given there are not. We present now correct calculations with a constant 5 instead of $(2\sqrt{2})$.

Proof. Let us assume that the theorem is false, so that we have with a constant 5 instead of $2(2)^{1/2}$. Then, for arbitrarily large n,

$$\left\| \frac{1}{f(x)} - \frac{P_n(x)}{Q_n(x)} \right\|_{L_\infty[0,\infty)} < 5^{-n\tau/(\rho\omega)}. \tag{2}$$

By assumption

$$\overline{\lim_{r\to\infty}} \frac{\log M(r)}{r^\rho} = \frac{\tau}{\omega}, \qquad \text{where} \quad M(r) = \operatorname{Max}_{|z|=r} |f(z)|. \tag{3}$$

From (3) it is easy to obtain (as before) that for each $\epsilon > 0$ and each $\delta > 1$ we have, for all $r \geq r_0(\epsilon, \delta) > 0$,

$$M(r\delta) \geq \{M(r)\}^{\frac{\delta^\rho \omega(1-\epsilon)}{\tau(1+\epsilon)}}. \tag{4}$$

Now for each $n \geq$ some n_0, we can find an $r > 0$ such that

$$f(r) = 5^{n\tau/(2\rho\omega)} \cdot 2. \tag{5}$$

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At this point r,

$$\frac{Q_n(r)}{P_n(r)} \leq 5^{\frac{n\tau}{2\rho\omega}} n. \tag{6}$$

for otherwise (2) would be contradicted. From (4) and (5) we get, with $\delta^\rho = 2\tau/\omega$,

$$f(r\delta) \leq \{f(r)\}^{\frac{2(1-\epsilon)}{(1+\epsilon)}} \leq 5^{\frac{n\tau(1-\epsilon)}{\rho\omega(1+\epsilon)}}. \tag{7}$$

On the other hand, since the coefficients of $P_n(x)$ and $Q_n(x)$ are nonnegative reals, we get by (6),

$$\frac{Q_n(r\delta)}{P_n(r\delta)} \leq \frac{\delta^n Q_n(r)}{P_n(r\delta)} \leq \left(\frac{2\tau}{\omega}\right)^{\frac{n}{\rho}} 5^{\frac{n\tau}{2\rho\omega}} n. \tag{8}$$

From (7) and (8), choosing ϵ to be sufficiently small, and observing that

$$5^{\frac{n\tau(1-3\epsilon)}{2\rho\omega(1+\epsilon)}} > (2n)\left(\frac{2\tau}{\omega}\right)^{\frac{n}{\rho}} \qquad \text{for all large } n.$$

we get

$$\begin{aligned} 5^{-\frac{n\tau}{\rho\omega}} &\leq 5^{-\frac{n\tau}{2\rho\omega}}(2\tau)^{-\frac{n}{\rho}} n^{-1}\omega^{\frac{n}{\rho}} - 5^{-\frac{n\tau(1-\epsilon)}{\rho\omega(1+\epsilon)}} \\ &\leq \frac{P_n(r\delta)}{Q_n(r\delta)} - \frac{1}{f(r\delta)}. \end{aligned} \tag{9}$$

Equation (9) clearly contradicts (2), and the result is proved.

Note. The Theorem is valid if, in (1), $2\sqrt{2}$ is replaced by any constant < 4.

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